Sequential Auctions with Synergy and Affiliation Across Auctions

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Abstract

This paper performs a structural analysis of sequential auctions with both synergy and affiliation across auctions. I propose a flexible yet tractable sequential auction model under the private value paradigm and establish its nonparametric identification, demonstrating an intuitive and general method for disentangling synergy from affiliation. After developing an estimation procedure closely tied to the identification steps, I apply it to data on adjacent oil and gas leases that are auctioned sequentially. I assess the role played by affiliation versus synergy in the observed allocation patterns and evaluate the counterfactual policy of bundled auctions.

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1 Introduction

On state trust lands in the New Mexico Permian Basin, adjacent oil and gas leases are commonly auctioned sequentially on the same day, and these adjacent leases are often won by the same bidder. This naturally raises the question of whether there is synergy (or complementarity) between leases and whether policy alternatives like bundled auctions could increase auction revenue and/or efficiency. The question is economically significant as the Permian Basin is one of the most prolific oil and gas basins in the world,[1] and leasing land is an important source of funding for the state's public institutions. Similar questions arise in sequential auctions of other objects, where positive synergy can arise due to economies of scale, increased market power, distribution of fixed costs, or complementarity of contiguous regions, and "negative" synergy can arise due to capacity constraints or close substitutes.[2]

While the question would benefit from structural analysis, sequential auctions linked by synergy present a challenge for modeling and identification. One reason is that equilibrium derivation is difficult for general sequential auction models.[3] As a result, sequential auctions depend on restrictive assumptions for analysis, leading to a struggle between tractability and capturing relevant features of the real-world auction. On the identification front, a difficulty lies in distinguishing synergy from affiliation of a bidder's values across auctions. With synergy, the fact that a bidder wins object 1 *causes* object 2 to be worth more to him; by contrast, affiliation across auctions means a bidder has similar values for objects 1 and 2 from the start.[4] Positive synergy tends to increase the revenue benefits of bundled auctions over separate auctions

[1] See www.britannica.com/place/Permian-Basin.

[2] A non-exhaustive list of examples includes spectrum auctions for AM and FM broadcasting licenses in New Zealand, cable TV license auctions in Israel (Gandal (1997)), auctions of agricultural tracts in the U.S. (Colwell and Yavaş (1994)), parcels in Singapore government auctions of residential and commercial land (Agarwal, Li, Teo, and Cheong (2017)), school milk procurement (Marshall, Raiff, Richard, and Schulenberg (2006)), truckload procurement for connecting origin-destination lanes (Mes (2008)), U.S. Bureau of Land Management auctions of mineral (oil, gas, coal) leases in 10+ states, USFS tracts of timber, construction procurement by state transportation departments including California, Oklahoma, Michigan, etc.

[3] To my knowledge, no one has characterized the set of equilibria for dynamically linked auctions with multi-unit demand and unrestricted joint distribution of values, even for the simplest case of two second-price auctions.

[4] Suppose a bidder's value for object 1 is v_1, his value for object 2 is v_2, and his value for both objects together is $v_1+v_2+\alpha$. Synergy refers to α being non-zero, while affiliation refers to correlation between v_1 and v_2. In this paper, I use the term "affiliation" to refer generally to a stochastically ordered form of correlation; this is weaker than and nests the mathematical definition of affiliation.

because bundling provides bidders a guarantee that the winner will realize that synergy (see Subramaniam and Venkatesh (2009) for simulations). Meanwhile, affiliation can decrease the revenue benefits of bundling by weakening the Schmalensee (1984) effect.[5] There are also cases where synergy recommends unbundling while affiliation recommends bundling, according to simulations.[6] So these are two distinct concepts that can lead to opposing policy recommendations, but in both cases the winner of object 1 will win object 2 with higher probability than other bidders. This observational similarity leads to an identification challenge analogous to that of structural state dependence versus persistent heterogeneity as highlighted by Heckman (1981). It also implies that failing to account for affiliation can lead to exaggerated estimates of synergy and vice versa.

This paper performs a structural analysis of sequential auctions with both synergy and affiliation across auctions to assess the counterfactual policy of bundling. I apply my analysis to pairs of adjacent leases in New Mexico, sold sequentially by a first-price auction followed by an English auction. I address the two challenges described above as follows. First, I propose a new model of sequential auctions that is flexible in the relationship between first- and second-auction values and functional form of synergy yet has a characterizable equilibrium. A key feature of this model is that the distribution of a bidder's second value is conditional on his first value at the time of the first auction. Second, auction data commonly provide bids of both winning and losing bidders, and I show that this information provides a basis for disentangling synergy and affiliation (or structural state dependence and persistent heterogeneity). The basic intuition is to ask whether bidders who bid similarly in the first auction diverge in subsequent behavior based on winning or losing that auction. This is a widely applicable identification strategy, which I illustrate with a regression discontinuity design and make formal in the main structural analysis.

To distinguish synergy and affiliation in a regression discontinuity design, I define the log difference between one's first-auction bid and highest competing bid thereof as the running variable. The running variable equaling zero marks the threshold between

[5]In Schmalensee (1984), bundling of items captures more surplus for the seller by reducing heterogeneity in buyer values. Affiliation weakens this effect because the extent to which bundling reduces heterogeneity declines with correlation of a bidder's values across items; in the extreme case of perfect correlation, bundling does not reduce heterogeneity at all but simply rescales the original value distribution.

[6]See simulated example in Online Appendix A.1.

winning and losing the first auction. A discontinuity in second-auction outcomes at the threshold indicates a discontinuity between bidders that just barely lost and just barely won the first auction, identifying the causal effect of winning – synergy – separately from affiliation.

To channel this intuition into a structural analysis, I model a sequence of auctions under the private value paradigm,[7] allowing both synergy and affiliation to take general functional forms. The timeline of my model proceeds as follows. At the first auction, bidders know their value for the first object, but their value for the second object might be uncertain or open to adjustment until the time of the second auction; the passage of time between auctions and events taking place therein can affect valuations. So bidders may not know their second value exactly at the time of the first auction, but they do know the distribution from which their second value will be drawn. To allow for affiliation, that distribution is conditional on their value for the first object. I place few restrictions on this conditional distribution, allowing a flexible relationship between a bidder's values for the first and second object. Bidders do learn their exact value for the second object at the beginning of the second auction. The distribution of second values being conditional on first values helps the model retain both flexibility and a characterizable equilibrium.

Meanwhile, bidders observe all bids and bidder identities from the first auction and hence know whether they won before bidding in the second auction. The bidder that won the first auction is affected by synergy, so his value going into the second auction is not just the stand-alone value of the second object, but a synergy-inclusive value. I define a synergy function that gives this synergy-inclusive value as a function of a bidder's stand-alone values for each object. Then in the first auction, bidders bid in light of not only their value for the first object, but also the expected benefit in the second auction from winning the first auction. Under some assumptions, I show that bids in the first auction are strictly increasing in a bidder's value for the first object, and there exists a unique Bayes-Nash equilibrium in the first auction.

I establish nonparametric identification of the model primitives from observable data. The primitives are the joint distribution of first-auction and second-auction values and the synergy function, while the observable data include all bids in the first auction, the transaction price in the second auction, and bidder identities. In particular, the synergy function is identified by comparing the second-auction value

[7]Justification for the private value paradigm is discussed in Section 2.

distributions of a first-auction winner and first-auction loser *conditional on the same first-auction bid*. This conditioning neutralizes affiliation and isolates the effect of synergy, since the first-auction winner benefits from synergy while the loser does not. The strategy is intuitive yet powerful enough to identify the synergy function nonparametrically. Closely following the identification steps, I develop a nonparametric multi-step estimation procedure that estimates the structural parameters of the auction model.

In the New Mexico auction data, I find both synergy and affiliation between leases, though affiliation is primarily responsible for the observed pattern in which the same bidder often wins adjacent leases. This result highlights the importance of allowing for affiliation across auctions. Counterfactual simulations reveal that bundled auctions would yield higher auction revenue than sequential auctions, on the order of 7 percent. They would also lead to a loss in allocative efficiency.

The paper's insights for modeling and identification extend beyond my empirical application. First, they are not unique to the first-price-then-second-price sequence observed in my data. I discuss extensions to two second-price auctions and to two first-price auctions in the Online Appendix. Second, objects in the sequence can have different covariates and be linked by positive or negative synergy; the model and identification argument do not restrict these. Third, the model extends to sequences longer than two, as I discuss in section 7.3. Finally, the basic insight of using similar bids to distinguish confounding elements is more broadly useful: the second event in the sequence need not be an auction. Suppose the causal effect of an acquisition (first event) on the acquirer's subsequent performance (second "event") needs to be disentangled from selection; high-bidding firms tend to be high-performing firms. If the acquisition procedure generates bids, they could be used to compare marginal winners and losers and isolate the causal effect.

The rest of the paper is organized as follows. The remainder of Section 1 provides a discussion of the related literature. Section 2 describes the data and empirical evidence. Section 3 develops the model. Section 4 establishes nonparametric identification of the model, while Section 5 develops an estimation procedure. Section 6 describes estimation details specific to the data at hand and discusses the estimation results. Section 7 performs counterfactual simulations of interest including those for bundled auctions. Section 8 concludes. The Online Appendix collects all proofs.

Related literature

This paper relates to three areas of literature: sequential auctions, synergy in auctions, and affiliation, which I discuss in turn. When bidders are unrestricted in the number of auctions they can win, a sequence of auctions is "sequential" only if they are dynamically linked for some reason, e.g. synergy in this paper. Examples in the theory literature include among others Ortega-Reichert (1968), Hausch (1986), Caillaud and Mezzetti (2004), and Benoit and Krishna (2001). This literature illustrates the difficulty of deriving equilibria for general sequential auctions. To make progress, papers restrict their analysis to two auctions and specialized models, e.g., a bidder's values for the two goods are the same, all bidders share the same values, bidders are represented by a single type variable, or values are independent across auctions and learned one at a time. Katzman (1999) and Lamy (2012) study sequential second-price auctions of two commodity goods. A bidder's value for his first unit of a commodity is the same regardless of which unit in the sequence he wins.

The empirical literature about sequential auctions begins with Ashenfelter (1989)'s study of wine auctions and includes among others Gandal (1997) and De Silva et al. (2005), whose regression analyses find evidence of synergy in real-world sequential auctions. Within that literature, structural econometric work on sequential auctions largely falls into two categories. The first category is sequential auctions of commodity goods as defined in the previous paragraph. Examples include Donald et al. (2006), Brendstrup and Paarsch (2006), Lamy (2010), and Donna and Espín-Sánchez (2018). In the second category, a bidder's value draws are independent across the sequence, conditional on his state. Dynamics arise because this state variable is affected by winning or entering previous auctions. Examples include Jofre-Bonet and Pesendorfer (2003) and Balat (2017) where the state variable captures bidders' capacity constraints, and Groeger (2014) where the state variable captures learning by doing in bid preparation. For clarity of comparison, these are sequential auctions with a form of synergy (broadly defined) but no affiliation.

There is also a literature on synergy in non-sequential auctions. Synergy is studied by Ausubel et al. (1997), Moreton and Spiller (1998), and Fox and Bajari (2013) among others in simultaneous ascending PCS auctions; Marshall et al. (2006) and Gentry et al. (2016) in simultaneous first-price auctions; and Cantillon and Pesendorfer (2006) in combinatorial first-price auctions.[8]

Finally, affiliated values in non-sequential auctions have been analyzed in theory and empirics by papers ranging from Milgrom and Weber (1982) and Pinkse and Tan (2005) to Laffont and Vuong (1996), Li, Perrigne, and Vuong (2002), Hubbard, Li, and Paarsch (2012), Li and Zhang (2015), Balat (2016), and Somaini (2018), among others. While these papers study affiliation across bidders within a single auction, I study affiliation across a sequence of auctions within bidder, a phenomenon that can confound measures of synergy unless properly distinguished.

2 Data

2.1 Overview

In oil and gas producing parts of the New Mexico State Trust Lands, the State Land Office (SLO) auctions leases for oil and gas development. Much of this leasing occurs in the Permian Basin, where knowledge of the geology is mature due to a long history of development dating back to the 1920s. Valuations of leases vary idiosyncratically among bidders; empirical evidence, discussed at the end of section 2.2, reveals the importance of a private component to bidders' values.

The terms of a lease, including its duration and production royalties due to the SLO, are made public well before its auction. While there is some variation, the amount of land most commonly covered by an oil and gas lease is a rectangle of 320 acres, or half a square mile.[9] Therefore, a section, which is a one square mile block, produces two such leases. Often, these two leases are auctioned on the same day. I will refer to two such leases as a "pair." The focus of study in this paper are pairs that were auctioned in the Permian Basin area during 2000-2014.

The SLO uses two auction formats, the first-price sealed-bid format and the English auction format. For pairs, the SLO has a convention of selling one of the leases by first-price sealed-bid and the other lease by English auction later in the day. The English auction always occurs later. Thus the two leases in a pair are auctioned in a

[8] Specifically, Marshall et al. (2006) model a constant synergy parameter in school milk auctions; Gentry et al. (2016) model synergy as a function of auction and bidder characteristics in highway procurement auctions; Cantillon and Pesendorfer (2006) model a free form of synergy to fit observed combinatorial bids in bus route auctions.

[9] The SLO prefers this size because it is long enough to allow horizontal drilling and is at least as large as the spacing units required for oil wells (40 acres) and gas wells (320 acres) by state rules.

sequence. In this paper, I refer to the earlier auction as the "first auction" and the later auction as the "second auction." To be clear, the two leases of a pair are not the only items being auctioned on a given day, nor are they auctioned back to back; in 2000-2014, the average number of Permian Basin leases auctioned on a single day was 39. In terms of observable data, I observe all bids and bidder identities for the first-price sealed bid auction. For the English auction, I observe the transaction price and the identity of the winner only; the data does not record the number of bidders. Table 1 displays the number of pairs observed by N, which is the number of bidders in the first-price sealed bid auction.

Table 2 displays within-pair statistics. The auction prices of paired leases are highly correlated, consistent with the geological similarity of adjacent leases. 93% of bidders winning the second auction ("A2") also participate in the first auction ("A1"). This is consistent with conversations with SLO staff; bidders interested in one half of a section are typically interested in the other half as well. Meanwhile, the probability that both leases in a pair will be won by the same bidder is higher than it would be if all A1 participants had an equal chance of winning A2. This shows that, at a simple correlation level, the winner of A1 is more likely to win A2 than other bidders.[10]

2.2 Evidence of synergy and affiliation

Why is the winner of A1 more likely to win A2? It is possible that the two leases are linked by synergy for various reasons. First, as noted in Sunnevåg (2000), adjacent leases can generate cost savings by reducing travel time and eliminating duplicates for equipment and crews. Second, in recent years much of the drilling in New Mexico has been horizontal; with permission from government authorities, adjacent leases can be put together to form a "project area" where horizontal wells can be drilled across lease borders.[11] For these and other reasons, there may be extra value to winning two adjacent leases beyond the sum of one's values for each lease individually. Intuitively, this synergy gives the winner of the first auction ("A1") a boost in winning the second auction ("A2"); the second lease becomes more valuable to him as a consequence of

[10]To check this correlation more formally, I perform a probit analysis in Online Appendix A.5, Table 8. In every specification, winning the first auction has a highly significant positive "effect" on the observed probability of winning the second auction.

[11]In the absence of permission, rules specify how far wells must be from unit boundaries. It is forbidden to access oil and gas outside lease boundaries.

winning the first.

However, the phenomenon could also be due to affiliation of a bidder's stand-alone values for the first (v_1) and second lease (v_2), in which a bidder with high v_1 is more likely to have a high v_2 to begin with. This concept is distinct from synergy as it concerns the correlation of values for individual tracts and has nothing to say about whether their sum is superadditive. If a firm views one tract favorably, it will probably view the adjacent tract favorably as well, given the similar location and geology. Not only can this lead to mismeasurement of synergy, an object of interest, but synergy and affiliation can also lead to opposing recommendations for auction design.

One way to explore the issue of synergy versus affiliation is to use a regression discontinuity design. For each bidder in the first auction, define

$$z \equiv ln(b) - ln(\text{highest competing } b)$$

where b is his bid in the first auction. Then $z > 0$ indicates an A1 winner, and $z < 0$ indicates an A1 loser. A large $|z|$ indicates a large gap between the first and second highest bids in A1. If bidders' v_1 and v_2 are affiliated, a larger $|z|$ makes it more likely that the same bidder will win both A1 and A2. On the other hand, if $|z|$ is very small, this means the A1 winner just barely won. In the absence of synergy, such a bidder should not be much more likely to win A2 than if he just barely lost. This is the idea I exploit to detect synergy; I look for a discontinuity in the probability of winning A2 at $z = 0$.[12] Formally, I seek to measure $\beta = y^+ - y^-$, where $y^+ \equiv lim_{z \to 0^+} E[y_i|z_i = z]$ and $y^- \equiv lim_{z \to 0^-} E[y_i|z_i = z]$. As proposed in Hahn, Todd, and Van der Klaauw (2001), I use local linear regression to estimate y^+ and y^-.

An RD plot of the data for the most frequent bidder is displayed in Figure 1. Two features of Figure 1 stand out. First, the probability of winning the second auction is increasing in z. This is consistent with affiliation of values, which makes a bidder's value for the first lease predictive of his value for the second lease. Second, there is a discontinuity at $z = 0$, consistent with synergy between adjacent tracts. The associated local linear regression results are shown in Table 3.[13] Columns (2)

[12] As an earlier example of exploiting the idea of RD in the auctions literature, Kawai and Nakabayashi (2014) examine bidders who narrowly won the first round of a multi-round auction, and find evidence of collusion in their persistent ranking in subsequent rounds.

[13] Figure 1 and Table 3 are obtained using the software packages described in Calonico, Cattaneo, and Titiunik (2014a). The second row of Table 3 corrects for the bias in conventional RD estimates

and (3) include fixed effects and interactions involving the number of bidders. There are suggestive indications of synergy, both in the plot of data and in the estimation results. The estimated jump in the probability of winning is roughly 0.2.

Figure 1 also suggests the presence of an idiosyncratic component to bidders' values. Note that all bidders learn all the bids submitted for the first lease before bidding on the second lease right next to it. Yet the probability of winning the second lease slopes upward steeply as a function of the first-auction bid, suggesting that idiosyncracies in valuation persist even after seeing all the other firms' bids.

Finally, Table 4 performs a probit analysis of winning A2, including as regressors z as defined above and an interaction of z with the number of auctions whose outcomes are realized between A1 and A2. The negative coefficient on the interaction says the number of intervening auctions attenuates the relationship between z and the probability of winning A2. This suggests that intervening auctions and time are a source of uncertainty and bidders may adjust their bids as the auction day proceeds.

3 A model of sequential auctions with synergy and affiliation across auctions

3.1 Setup

A pair of objects is sold via auction. One object is sold by a first-price sealed-bid auction, and the other is sold by an English auction, which happens later chronologically. There are N ex-ante symmetric bidders. Before bidding in the first auction, each bidder draws a value $v_1 \sim F_1(\cdot)$ which is his stand-alone, private value for the object in the first auction.

Between the first auction (A1) and second auction (A2), there might be noise that affects bidders' values for the second object. I elaborate on this later. Therefore, bidders may not know their stand-alone value for the second object (v_2) with certainty at the time of the first auction. However, they do know the distribution from which v_2 will be drawn. That distribution is conditional on v_1 to allow for affiliation between v_1 and v_2: $v_2 \sim F_2(\cdot|v_1)$. A special case would be that $E[v_2|v_1] = v_1$, but the model is more general. Another special case is that $F_2(\cdot|v_1)$ has arbitrarily small variance or

as discussed in Calonico, Cattaneo, and Titiunik (2014b), and the third row increases the standard error to account for the fact that this bias is itself estimated.

is degenerate, so that v_2 is known with certainty at A1. For non-degenerate $F_2(\cdot|v_1)$, the exact value of v_2 is learned after the first but before the second auction.

The bidder that won the first auction benefits from synergy, so his value going into the second auction is not just the stand-alone value v_2 but a synergy-inclusive value which I define as $s(v_1, v_2)$. Positive synergy means $s(v_1, v_2) > v_2$; "negative synergy" means $s(v_1, v_2) < v_2$. I allow the synergy function $s(\cdot, \cdot)$ to be a function of both v_1 and v_2 to be as general as possible. The size of synergy is private to each bidder, since it is a function of v_1 and v_2, which are different for each bidder and private information. For convenience of notation I define the distribution of $s(v_1, v_2)$ conditional on v_1 as follows: $D(x|v_1) \equiv \Pr(s(v_1, v_2) \leq x|v_1)$. So winners of A1 draw their synergy-inclusive value for the second object from the distribution $D(\cdot|v_1)$. I assume that the same set of bidders participate in the first and second auction.

Now I discuss the ideas underlying this model. I do not explicitly model the source of noise between the first and second auction as sources can vary. As evidenced in Table 4, one source of noise is other auctions that take place in between the two sales that affect bidders' valuations by the time of the second auction. When the sequence is spread out in time, noise can come from the passage of time and events taking place therein.[14]

The distribution of v_2 is conditional on v_1, but it is not conditional on any other private information. The underlying assumption is that v_1 is a sufficient statistic for any private, bidder-specific info known at the time of A1 that a bidder's $F_2(\cdot|\cdot)$ could depend on. The assumption is reasonable when the objects are related in a way that determinants of private value "shocks" are similar. This is a distinct notion from observable similarity; objects in A1 and A2 may have different descriptive covariates. In contexts where this does not apply, the alternative is to have a separate private signal for the second object at the time of the first auction. However, general multi-dimensional types with one bid at a time raise issues for characterizing equilibrium.

As I derive equilibrium bids in this model, I make the following assumptions.

AS1 (v_1, v_2) are independent across bidders.

AS2 $F_1(\cdot)$ is differentiable, with positive, finite, and continuous density $f_1(\cdot)$.

[14]In Marshall et al. (2006), school milk procurements take place from May through August of each year, and in Gandal (1997), Israeli cable TV licenses are auctioned over a period spanning 1988-1991.

AS3 $F_2(\cdot|\cdot)$ and $D(\cdot|\cdot)$ are differentiable in both arguments, with positive, finite densities $f_2(\cdot|\cdot)$ and $d(\cdot|\cdot)$ continuous in both arguments, and have the same compact support $\forall v_1$.

AS4 $F_2(\cdot|v_1)$ is stochastically ordered in v_1; i.e. for $v_1' > v_1$, $F_2(x|v_1') \leq F_2(x|v_1)$ $\forall x$.

AS5 $|E[v_2|v_1] - E[v_2|v_1']| \leq |v_1 - v_1'|$

AS6 $\frac{\partial s(v_1,v_2)}{\partial v_1} \geq 0$ and $\frac{\partial s(v_1,v_2)}{\partial v_2} \geq 0$.

AS1 means that while values can be affiliated across auctions, they are independent across bidders. AS3, which says all bidders bidding in the second auction draw their values from the same support, is an assumption included for completeness as it provides for full identification of the value distributions. It is not a critical assumption in the sense that, if the supports are different, the value distributions will be identified only where the supports overlap.[15] AS4 defines precisely what I mean by "affiliation" in this paper; it means that a bidder with higher v_1 is weakly more likely to have a higher v_2. This assumption speaks to correlation between the stand-alone values of items 1 and 2 and is unrelated to synergy, i.e. whether $s(v_1, v_2) > v_2$. It is used in establishing monotonic bidding in the first auction, along with the remaining two assumptions that follow. AS5 resembles a Lipschitz condition and rules out extreme movements or divergence of the expected value of v_2 as a function of v_1. This assumption is easy to verify in data once $F_2(\cdot|\cdot)$ is estimated. AS6 says that $s(v_1, v_2)$, the synergy-included value of the second object, is a nondecreasing function of v_1 and v_2. This does not rule out "negative synergy", or $s(v_1, v_2) < v_2$. Rather, it means that if a firm's stand-alone value for an object increases, the firm's synergy-included value for the object does not decrease, all else equal.

3.2 Bidding in the sequential auctions

Working backwards, I discuss bidding in the second auction before going to the first auction. The second auction is an English auction. Under the private value paradigm,

[15] Online Appendix A.1 provides further discussion.

it is weakly dominated for a bidder to drop out below his value for the object. For the bidder who won the first auction, this value is $s(v_1, v_2)$. For all other bidders, this value is v_2. For the sake of providing intuition, the explanatory text takes the perspective of positive synergy; however, the derived results do not depend on synergy being positive.

Before moving on to the first auction, it is useful to introduce some notation that simplifies the expected profit function for bidders. Though the model is different, I follow the notation used by Lamy (2012). The distribution of the highest competing bid a bidder faces in the second auction given that he wins the first auction and the highest competing bid there is t is

$$H_1(u|t) = \tilde{F}_2(u|b \leq t)^{N-2}\tilde{F}_2(u|b = t). \tag{1}$$

To explain, the probability that the highest competing bid he faces in A2 is less than u is equal to the probability that all bidders other than him have values less than u for the second item. Since the highest competing bid in A1 is t, the other bidders in A2 consist of one bidder who bid t in A1 and $N-2$ bidders who bid less than t in A1. The right-hand side of (1) expresses the probability that all of these competing bidders have values less than u. The subscript 1 on $H(\cdot|\cdot)$ indicates the bidder won the first auction.

Next, the distribution of the highest competing bid a bidder faces in A2 given that he loses A1 and the highest competing bid there is t is

$$H_2(u|t) = \tilde{F}_2(u|b \leq t)^{N-2}\tilde{D}(u|b = t). \tag{2}$$

The subscript 2 on $H(\cdot|\cdot)$ indicates the bidder lost the first auction. The right-hand side of (2) is the same as that of (1) except that $\tilde{D}(u|b = t)$ replaces $\tilde{F}_2(u|b = t)$. Having lost A1, the bidder knows he will be competing against the winner of A1, who benefits from synergy. Therefore, $H_2(\cdot|\cdot)$ is different from $H_1(\cdot|\cdot)$ if synergy exists.

Now I consider bidding in the first auction (A1), which is a first-price sealed-bid auction, supposing for now a symmetric equilibrium. Let $G(\cdot)$ be the distribution of bids in this auction, so that $G^{N-1}(\cdot)$ is the distribution of the highest bid out of $N-1$ bidders. The expected profit from the two auctions at the time of the first auction, where the bidder bids b is

$$
\begin{aligned}
\pi(v_1, b) \;\; = \;\; & \int\limits_{v_2=\underline{v}}^{\bar{v}} \Bigg\{ \int\limits_{t=\underline{b}}^{b} \Big(v_1 - b + \int\limits_{u=\underline{v}}^{s(v_1,v_2)} (s(v_1, v_2) - u) dH_1(u|t) \Big) dG^{N-1}(t) \\
& + \int\limits_{t=b}^{\bar{b}} \int\limits_{u=\underline{v}}^{v_2} (v_2 - u) dH_2(u|t) dG^{N-1}(t) \Bigg\} dF_2(v_2|v_1).
\end{aligned}
$$

The outer integral over v_2 reflects potential uncertainty over v_2 at the time of the first auction. The first expression inside the outer integral represents the case where the bidder wins A1, and the second expression represents the case where the bidder loses A1. Notice that when he wins A1, he benefits not only from $v_1 - b$ but also from two facts: 1) the second item is now worth $s(v_1, v_2)$ to him rather than just v_2 due to synergy, and 2) none of his competitors in A2 have this synergy, resulting in weaker competition than otherwise, i.e. $H_1(\cdot|\cdot)$ versus $H_2(\cdot|\cdot)$.

A bidder will bid the b that maximizes his expected profit $\pi(v_1, b)$. Taking the derivative of $\pi(v_1, b)$ with respect to b and setting it equal to zero gives the first-order condition. Using integration by parts and rearranging, the first-order condition can be simplified to (3).

$$
b = v_1 + \underbrace{\int\limits_{v_2=\underline{v}}^{\bar{v}} \Bigg\{ \int\limits_{u=\underline{v}}^{s(v_1,v_2)} H_1(u|b)du - \int\limits_{u=\underline{v}}^{v_2} H_2(u|b)du \Bigg\} dF_2(v_2|v_1)}_{\text{expected benefit in A2 of winning A1}} - \frac{G(b)}{(N-1)g(b)}. \qquad (3)
$$

It is instructive to compare this first-order condition to that of a stand-alone first-price auction. From Guerre, Perrigne, and Vuong (2000), the first-order condition for a stand-alone first-price auction is $b = v_1 - \frac{G(b)}{(N-1)g(b)}$. In (3), there is an additional term on the right-hand side that represents the expected benefit in the second auction from winning the first auction. Namely, winning the first auction increases a bidder's value for the second object from v_2 to $s(v_1, v_2)$ and weakens competition by preventing synergy for other bidders. So v_1 in the usual first-order condition is replaced by v_1 plus the expected benefit of synergy in my model. If there is no synergy for bidders, i.e. $s(v_1, v_2) = v_2$, then (3) collapses to $b = v_1 - \frac{G(b)}{(N-1)g(b)}$, as in Guerre et al. (2000).

3.3 Equilibrium properties

Given AS1-AS6, I show that it is impossible for a strictly lower first-auction bid to be a best response for a strictly higher value.[16] Also, the right-hand side of (3) is strictly increasing in v_1, so a single bid cannot be a best response for two different values. A proof along these lines leads to the following proposition.

Proposition 1. *In any symmetric equilibrium, the bid function $b(v_1)$ in the first auction is strictly increasing in v_1.*

Since the bid function is strictly increasing, it is invertible in a given equilibrium. Functions of v_1 can be converted to functions of b. For instance, $F_2(v_2|v_1 = x) = \tilde{F}_2(v_2|b = b(x))$ and $\tilde{F}_2(v_2|b)$ retains the stochastic ordering property of $F_2(v_2|v_1)$. Similarly, if I define a new function $\tilde{s}(\cdot,\cdot)$ such that $\tilde{s}(b(v_1), v_2) = s(v_1, v_2)$, $\tilde{s}(\cdot,\cdot)$ retains the weak monotonicity of $s(\cdot,\cdot)$. Replacing $F_2(v_2|v_1)$ with $\tilde{F}_2(v_2|b)$ and $s(v_1, v_2)$ with $\tilde{s}(b, v_2)$ in (3) and rearranging defines the inverse bid function:

$$\xi(b) \equiv b + \frac{G(b)}{(N-1)g(b)} - \int_{v_2=\underline{v}}^{\bar{v}} \left\{ \int_{u=\underline{v}}^{\tilde{s}(b,v_2)} H_1(u|b)du - \int_{u=\underline{v}}^{v_2} H_2(u|b)du \right\} d\tilde{F}_2(v_2|b) = v_1. \quad (4)$$

Likewise, functions of b can be converted to functions of v_1. In particular, $H_1(u|b)$ and $H_2(u|b)$ can be replaced with $\tilde{H}_1(u|\xi(b))$ and $\tilde{H}_2(u|\xi(b))$, and $G(b)$ can be replaced with $F_1(\xi(b))$. After some algebra, this gives the following differential equation that must be satisfied in equilibrium:

$$\frac{dP(v_1)}{dv_1} = T(v_1)\frac{d}{dv_1}[F_1(v_1)^{N-1}], \quad (5)$$

where $P(v_1) \equiv b(v_1)F_1(v_1)^{N-1}$ is the bidder's expected payment and

$$T(v_1) \equiv v_1 + \underbrace{\int_{v_2=\underline{v}}^{\bar{v}} \left\{ \int_{u=\underline{v}}^{s(v_1,v_2)} \tilde{H}_1(u|v_1)du - \int_{u=\underline{v}}^{v_2} \tilde{H}_2(u|v_1)du \right\} dF_2(v_2|v_1)}_{\text{expected benefit in A2 of winning A1}}.$$

[16] What I show here implies that the individually rational tieless single crossing condition as defined in Reny and Zamir (2004) is satisfied.

Equation (5) is similar to the equilibrium condition for a stand-alone first-price auction as studied in Riley and Samuelson (1981), except that $T(v_1)$ takes the place of what was v. Solving this differential equation leads to Proposition 2.

Proposition 2. *There is a unique symmetric Bayes-Nash equilibrium for the first auction, given by* $b(v_1) = \int_{\underline{v}}^{v_1} T(x)dF_1(x)^{N-1}/F_1(v_1)^{N-1}$ *and* $b(\underline{v}) = T(\underline{v})$*. The bidder's expected payment is* $\int_{\underline{v}}^{v_1} T(x)dF_1(x)^{N-1}$.

If there is no synergy, $T(v_1) = v_1$; if synergy is positive, $T(v_1) > v_1$, and if it is negative, $T(v_1) < v_1$. This means the bidder's expected payment $\int_{\underline{v}}^{v_1} T(x)dF_1(x)^{N-1}$ and auction revenue in the first auction are higher (lower) when synergy is positive (negative) than when synergy is zero; i.e. there is a synergy premium.

3.4 Risk aversion

The model of sequential auctions with synergy can also be extended to the case where bidders are risk-averse. Since the second auction (A2) is an English auction, it remains weakly dominated for a bidder to drop out below his value, regardless of risk aversion. However, risk aversion does affect bidding in the first auction (A1), which uses the first-price sealed-bid format. Also, risk aversion changes the extent to which the second auction affects bidding in the first auction.

Let $U(\cdot)$ be a twice continuously differentiable utility function with $U(0) = 0$, $U'(\cdot) > 0$, and $U''(\cdot) \leq 0$. Expected profit at the time of the first auction is

$$\begin{aligned}\pi(v_1, b) \;=\; & \int_{v_2=\underline{v}}^{\bar{v}} \Bigg\{ \int_{t=\underline{b}}^{b} \int_{u=\underline{v}}^{s(v_1,v_2)} U(v_1 - b + s(v_1, v_2) - u) dH_1(u|t) dG^{N-1}(t) \\ & + U(v_1 - b) \int_{t=\underline{b}}^{b} \int_{u=s(v_1,v_2)}^{\bar{v}} dH_1(u|t) dG^{N-1}(t) \\ & + \int_{t=b}^{\bar{b}} \int_{u=\underline{v}}^{v_2} U(v_2 - u) dH_2(u|t) dG^{N-1}(t) \Bigg\} dF_2(v_2|v_1). \end{aligned} \tag{6}$$

All profits now show up inside the utility function $U(\cdot)$. The first expression inside the outer integral represents the case where a bidder wins both auctions, the second expression is the case of winning only the first auction, and the third expression is the case of winning only the second auction.

From the perspective of bidders in the first auction, the second auction is like a lottery. When bidders are risk-averse, this lottery has a lower certainty equivalent and is discounted relative to the case of risk neutrality. Thus, as bidders grow more risk-averse, the second auction matters less; at very high levels of risk aversion, bidding in A1 converges to that of a stand-alone first-price auction.

Risk aversion can lead to higher or lower revenue in A1 compared to the risk neutral case, as there are two opposing forces. On the one hand, risk aversion pushes bidders to bid more in a first-price auction because they want to buy insurance against the possibility of losing. On the other hand, risk aversion causes bidders to discount uncertain payoffs from A2 when they bid in A1 (as discussed in the previous paragraph), and this decreases the synergy premium in A1 bids.

4 Identification

In this section, I show that the model primitives, meaning the value distributions $F_1(\cdot)$, $F_2(\cdot|\cdot)$, and the synergy function $s(\cdot,\cdot)$, are identified from observable data, which are the joint distribution of first auction bids and second auction prices, along with bidder identities. The intuition behind this identification result is as follows: suppose we observe two ex-ante symmetric bidders submit identical bids in the first auction, but one of them wins and the other loses. The fact that they bid the same means they had the same v_1. If winning the first auction has no effect on bidders' values for the second item, the winner should behave no differently than the loser in the second auction. By comparing the behavior of the winner and the loser, I can measure the synergy that comes from having both objects. Online Appendix A.2 provides extensions to asymmetric bidders, a sequence of two second-price auctions, and a sequence of two first-price auctions.

4.1 Identification

The identification strategy begins by looking at the second auction, and then proceeds back to the first auction.

Proposition 3. *$F_1(\cdot)$, $F_2(\cdot|\cdot)$, and $s(\cdot,\cdot)$ are identified from all the bids in the first auction, the transaction price in the second auction, and bidder identities. They can be recovered by following these steps: (i) The value distributions involved in the second auction, $\tilde{F}_2(\cdot|b)$ and $\tilde{D}(\cdot|b)$ are identified from observables. (ii) Once $\tilde{F}_2(\cdot|b)$ and $\tilde{D}(\cdot|b)$ are known, the synergy function $\tilde{s}(b,\cdot)$ is nonparametrically identified. (iii) Using $\tilde{F}_2(\cdot|b)$ and $\tilde{s}(b,\cdot)$, $F_1(\cdot)$ is identified nonparametrically from bids in the first auction. (iv) $\tilde{F}_2(\cdot|b)$ and $\tilde{s}(b,\cdot)$ can be converted to $F_2(\cdot|v_1)$ and $s(v_1,\cdot)$.*

The identification argument for step (i), presented in the proof, is based on Athey and Haile (2002). In their Theorem 2, Athey and Haile (2002) show that the value distributions of asymmetric IPV bidders are identified from transaction prices and winner identities. When it comes to the second auction in the model, the first auction induces asymmetry between bidders that were ex-ante symmetric. Specifically, the winner $w1$ of the first auction draws his second value from $\tilde{D}(\cdot|b_{w1})$, and each loser i from the first auction draws from $\tilde{F}_2(\cdot|b_i)$. For a fixed set of first-auction bids $\{b_i\}$, we can apply Theorem 2 of Athey and Haile (2002), so each of these distributions is identified from transaction prices and winner identities in the second auction.

Step (ii) says that having identified $\tilde{F}_2(\cdot|b)$ and $\tilde{D}(\cdot|b)$, the synergy function $\tilde{s}(\cdot,\cdot)$ is also identified. As mentioned at the beginning of the identification section, the intuition is to compare how a first-auction winner and first-auction loser behave differently in the second auction when they are otherwise identical, even to the point of having the same v_1. We can do just this by comparing $\tilde{F}_2(\cdot|b)$ and $\tilde{D}(\cdot|b)$; by conditioning on $b(v_1)$, we compare two bidders who only differ in that one of them won the first auction while the other did not. Therefore, the difference between $\tilde{F}_2(\cdot|b)$ and $\tilde{D}(\cdot|b)$ can be attributed to synergy. More precisely, recall that $\tilde{F}_2(\cdot|b)$ is the distribution of $v_2|b$ and $\tilde{D}(\cdot|b)$ is the distribution of $\tilde{s}(b,v_2)|b$. Since $\tilde{s}(b,v_2)$ is weakly increasing in v_2, $\tilde{s}(b,\cdot)$ must map the α-quantile of $\tilde{F}_2(\cdot|b)$ to the α-quantile of $\tilde{D}(\cdot|b)$. Since $\tilde{F}_2(\cdot|b)$ and $\tilde{D}(\cdot|b)$ are identified, this mapping provides for nonparametric identification of $\tilde{s}(\cdot,\cdot)$. Figure 2 illustrates the idea graphically; the function $\tilde{s}(b,\cdot)$ maps the origin of each arrow to the destination of that arrow.

In step (iii), having identified $\tilde{F}_2(\cdot|b)$ and $\tilde{s}(b,\cdot)$, $F_1(\cdot)$ can be identified using the inverse bid function (4) for the first auction. The inverse bid function can be computed at this stage because its components - $\tilde{F}_2(\cdot|b)$ and $\tilde{s}(b,\cdot)$ as well as the observed bid distribution $G(b)$ - are identified. Since bids $b(v_1)$ are monotonic in v_1, any quantile of v_1 can be recovered by computing the inverse bid function for that quantile of b, and this recovers $F_1(\cdot)$ nonparametrically.

Finally, step (iv) ties the remaining loose ends. Denote the α-quantile of v_1 and b by $v_1(\alpha)$ and $b(\alpha)$, respectively. Then $F_2(v_2|v_1(\alpha)) = \tilde{F}_2(v_2|b(v_1(\alpha))) = \tilde{F}_2(v_2|b(\alpha))$, and $s(v_1(\alpha), v_2) = \tilde{s}(b(v_1(\alpha)), v_2) = \tilde{s}(b(\alpha), v_2)$. Now all the primitives of the model, $F_1(\cdot)$, $F_2(\cdot|\cdot)$, and $s(\cdot,\cdot)$, are identified.

The logic of the identification argument reveals that, for purposes of disentangling synergy and affiliation, the exact bid function in the first auction need not be known. Neither need we know the bidders' utility function. Knowledge of the bid function's monotonicity is sufficient for steps (i) and (ii) above. Of course, identification of $F_1(\cdot)$ (step (iii)) would no longer be possible, but in some applications identifying synergy is the main object of interest. What can we say about identification if the synergy function $s(v_1, v_2)$ is stochastic? The stochastic component of synergy is not separately identified, i.e. data generated by such a model is fully rationalized by a model with a deterministic synergy function, namely the one constructed by following the identification steps above.

4.2 Identification with risk aversion

Steps (i) and (ii) in Proposition 3 apply even with risk-averse bidders, since bidding strategies in the second (English) auction are unaffected by risk aversion. This means $\tilde{F}_2(\cdot|b)$ and $\tilde{s}(b,\cdot)$ are identified regardless of risk aversion. The functions $U(\cdot)$ and $F_1(\cdot)$ remain to be identified.

Replacing $s(v_1, v_2)$ with $\tilde{s}(b, v_2)$ and $F_2(v_2|v_1)$ with $\tilde{F}_2(v_2|b)$ in the first-order condition for risk-averse bidders yields

$$\begin{aligned}\frac{G(b)}{(N-1)g(b)} \quad = \quad & \textstyle\int_{v_2=\underline{v}}^{\bar{v}}\{\int_{u=\underline{v}}^{\tilde{s}(b,v_2)} U(v_1 - b + \tilde{s}(b,v_2) - u)dH_1(u|b) \\ & + \textstyle\int_{u=\tilde{s}(b,v_2)}^{\bar{v}} U(v_1 - b)dH_1(u|b) - \int_{u=\underline{v}}^{v_2} U(v_2 - u)dH_2(u|b)\}d\tilde{F}_2(v_2|b)/ \\ & \textstyle\int_{v_2=\underline{v}}^{\bar{v}}\{\int_{u=\underline{v}}^{\tilde{s}(b,v_2)} U'(v_1 - b + \tilde{s}(b,v_2) - u)dH_1(u|t \le b) \\ & + \textstyle\int_{u=\tilde{s}(b,v_2)}^{\bar{v}} U'(v_1 - b)dH_1(u|t \le b)\}d\tilde{F}_2(v_2|b).\end{aligned} \tag{7}$$

Every term on the right-hand side is observed or identified except for v_1 and $U(\cdot)$. Meanwhile, with $U'(\cdot) > 0$ and $U''(\cdot) \le 0$ under risk aversion, the right-hand side is strictly increasing in v_1. This means that if I know $U(\cdot)$, I can use this first-order condition to uniqely back out the v_1 associated with any bid b. The only remaining piece then is to identify $U(\cdot)$, or the risk aversion level.

If the stand-alone value of auction items, conditional on having the same characteristics, should have similar distributions regardless of placement in the auction sequence,[17] this can be used to identify the risk aversion parameter ρ, in the spirit of Lu and Perrigne (2008). The unconditional distribution of stand-alone values in A2, given by $F_2(v_2) = \int F_2(v_2|b)dG(b)$, is identified independently of ρ. Meanwhile, $\xi(b, \rho)$, the inverse bid function for A1 defined by (7), does depend on ρ. The risk aversion parameter is identified if there is a unique value of ρ that satisfies the appropriate criterion of similarity between the distribution $F_2(\cdot)$ and the distribution of $\xi(b, \rho)$ for $b \sim G(b)$. In particular, we know from auction theory that bidders at the bottom of the value distribution bid without markup. Therefore, of the two opposing effects of risk aversion discussed at the end of section 3, only the second one – the decrease in the certainty equivalent value of A2 – is operative at $v_1 = \underline{v}$. As a result, $\xi(\underline{b}, \rho)$ is monotonic in ρ, and there is a unique ρ that satisfies $\xi(\underline{b}, \rho) = F_2^{-1}(0)$.

5 Estimation

5.1 A multi-step estimation procedure

I develop a multi-step estimation procedure that closely follows the identification steps. Following the identification strategy in section 4.1, the first step estimates $\tilde{D}(\cdot|b)$ and $\tilde{F}_2(\cdot|b)$, which are the distributions of second auction values for the first-auction winner and loser, respectively, conditional on the first-auction bid. For this task I use a sieve maximum likelihood estimator with Bernstein polynomial bases.[18]

Specifically, in the second auction I observe for each item the transaction price p, the identity of the winner, and the identity and bids of all bidders in the related first auction. Taking the case of $N = 2$ (two bidders in the first auction) as an expositional example, let $\tilde{d}(\cdot|\cdot)$ and $\tilde{f}_2(\cdot|\cdot)$ be the derivatives with respect to the first argument of $\tilde{D}$ and $\tilde{F}_2$, and let b_{w1} and b_{l1} be the first-auction bids of the winner and

[17] In Online Appendix A.5 I check that there is no statistically significant difference in post-auction production between A1 and A2 leases.

[18] General properties of sieve estimators, including sieve maximum likelihood, are discussed in Chen (2007). Bierens and Song (2012) is another example of sieve estimation for auctions, and Komarova (2017) is an example of using Bernstein polynomials in particular for the ascending auction framework.

loser, respectively. The likelihood of the second-auction price and winner given the first-auction data can be expressed as follows for each item.

If the first-auction winner wins the second auction, it is $L = (1-\tilde{D}(p|b_{w1}))\tilde{f}_2(p|b_{l1})$. If the first-auction loser wins the second auction, it is $L = (1-\tilde{F}_2(p|b_{l1}))\tilde{d}(p|b_{w1})$. The log-likelihood of the second-auction data as a whole is the sum of all the k item-level log-likelihoods, $\mathscr{L} = \frac{1}{k}\sum_{i=1}^{k} log(L_i)$.

Now, to use sieve estimation, $\tilde{D}(\cdot|\cdot)$ and $\tilde{F}_2(\cdot|\cdot)$ are approximated with Bernstein polynomials. Specifically, $\tilde{D}(v|b)$ and $\tilde{F}_2(v|b)$ are approximated by bivariate Bernstein polynomials of the form

$$B(v,b) \equiv \sum_{i=0}^{m}\sum_{j=0}^{n}\gamma_{i,j}\begin{pmatrix} m \\ i \end{pmatrix} v^i(1-v)^{m-i}\begin{pmatrix} n \\ j \end{pmatrix} b^j(1-b)^{n-j}, \tag{8}$$

where m and n are the polynomial degrees for v and b, respectively. This approximation does place a restriction that $\tilde{D}(v|b)$ and $\tilde{F}_2(v|b)$ be continuous in b. Finally, $\tilde{D}(\cdot|\cdot)$ and $\tilde{F}_2(\cdot|\cdot)$ are estimated by finding the polynomial parameters γ that maximize the log likelihood $\mathscr{L}$. A benefit of using Bernstein polynomials is that it is easy to impose required properties. Since $\tilde{D}(\cdot|b)$ and $\tilde{F}_2(\cdot|b)$ are cdfs, I restrict $B(v,b)$ to be weakly increasing in v by applying the restriction $\gamma_{i,j} \leq \gamma_{i',j}$ if $i < i'$. I also impose $\gamma_{0,j} = 0$ (i.e. $F_2(\underline{v}|b) = 0$) and $\gamma_{m,j} = 1$ (i.e. $F_2(\bar{v}|b) = 1$).

The second step of the estimation procedure is to estimate $\tilde{s}(b, v_2)$. As the proof of identification for $\tilde{s}(b, v_2)$ is constructive, I use it directly as an estimator as follows. For a fixed b, $\tilde{s}(b,\cdot)$ maps the α-quantile of $\tilde{F}_2(\cdot|b)$ to the α-quantile of $\tilde{D}(\cdot|b)$, because $\tilde{s}(b, v_2)$ is monotonic in v_2. Therefore, given $\hat{\tilde{F}}(\cdot|\cdot)$ and $\hat{\tilde{D}}(\cdot|\cdot)$ from the first step of the estimation procedure, I obtain $\hat{\tilde{s}}(b,\cdot)$ nonparametrically as the function that maps $\hat{\tilde{F}}_2^{-1}(\alpha|b)$ to $\hat{\tilde{D}}^{-1}(\alpha|b)$ for every quantile α on a grid over [0,1]; that is, $\hat{\tilde{s}}(b,\cdot) = \hat{\tilde{D}}^{-1}(\hat{\tilde{F}}_2(\cdot|b)|b)$. Since I can repeat this procedure for any b I choose, I have an estimator for $\tilde{s}(\cdot,\cdot)$.

The third step of the estimation procedure is to estimate $F_1(\cdot)$, the distribution of v_1, using the inverse bid function $\xi(b)$ derived in (4):

$$\hat{v}_1 = \hat{\xi}(b) \equiv b + \frac{\hat{G}(b)}{(N-1)\hat{g}(b)} - \int_{v_2=\underline{v}}^{\bar{v}}\left\{\int_{u=\underline{v}}^{\hat{\tilde{s}}(b,v_2)}\hat{H}_1(u|b)du - \int_{u=\underline{v}}^{v_2}\hat{H}_2(u|b)du\right\}d\hat{\tilde{F}}_2(v_2|b).$$

The distribution and density of first-auction bids $\hat{G}(\cdot)$ and $\hat{g}(\cdot)$ can be estimated from observed bids nonparametrically, while $\hat{\hat{s}}(\cdot,\cdot)$ and $\hat{\hat{F}}_2(\cdot|\cdot)$ are known from estimation steps 1 and 2. The distributions $\hat{H}_1(\cdot|\cdot)$ and $\hat{H}_2(\cdot|\cdot)$, defined in (1) and (2), are functions of $\hat{\hat{F}}(\cdot|\cdot)$ and $\hat{\hat{D}}(\cdot|\cdot)$. Therefore, I can compute $\hat{\xi}(b)$. Since bids are monotonic in v_1, we have $F_1^{-1}(\alpha) \equiv v_1(\alpha) = \xi(b(\alpha))$ for any quantile α. Upon computing $\hat{\xi}(b(\alpha))$ for a grid of α over [0,1], I obtain $\hat{F}_1(\cdot)$ as the function that maps $\hat{\xi}(b(\alpha))$ to α. I prove the consistency of the estimators for $F_2(\cdot|\cdot)$, $D(\cdot|\cdot)$, $s(\cdot,\cdot)$, and $F_1(\cdot)$ in section A.3 of the Online Appendix. In Online Appendix A.4, I conduct Monte Carlo studies to evaluate the performance of the estimator in finite samples, compare selection criteria for sieve orders,[19] and assess the coverage rate of bootstrap percentile intervals.

If bidders are risk-averse, $\hat{F}_1(\cdot)$ must be estimated conditional on a utility function, since the first-order condition for bidding depends on it. I assume a parametric utility function characterized by a risk aversion parameter ρ. I develop an estimator for ρ based on identification using the auction sequence, as explained in section 4.2. The criterion for similarity between $F_1(\cdot)$ and $F_2(\cdot)$ could be that the distance between the two distributions be minimized, leading to the minimum distance estimator

$$\hat{\rho} = \arg\min_{\rho} \sum_{\alpha} [\hat{\xi}(b(\alpha), \rho) - \hat{F}_2^{-1}(\alpha)]^2. \tag{9}$$

5.2 Auction covariates

The ideal way to deal with auction heterogeneity would be to estimate separate value distributions for every value of vector z, but this leads to a curse of dimensionality. As a result, a common approach in the empirical auction literature, as explained in Haile, Hong, and Shum (2003), is to homogenize bids across auctions by "demeaning" them, i.e. transforming bids to regression residuals $\eta = b - z'\beta$ and working with the residuals in estimation. This allows one to "pool" all the data. The underlying assumptions are that $v = z'\beta + \epsilon$ (additive separability) and that the distribution of ϵ is invariant to z (independence).

Instead of transforming bids and prices to demeaned residuals, I transform bids and prices to quantiles conditional on $z'\beta$; that is, b becomes $\tilde{b} \equiv G(b|z'\beta)$ and p

[19] I select sieve orders by minimizing the criterion $0.5k - 2\ln(L)$, where k is the number of parameters and $ln(L)$ is the log likelihood of the data.

becomes $\tilde{p} \equiv J(p|z'\beta)$, where $G(\cdot)$ is the distribution of first auction bids and $J(\cdot)$ is the distribution of second auction prices. Both $G(\cdot)$ and $J(\cdot)$ are observed in the data.[20] I then use these quantiles to perform the first step of estimation. Afterwards, the output from this step is transformed back to real values before proceeding with the other steps of estimation. Note that demeaning is a special case of taking quantiles; under assumptions of additive separability and independence, the residuals $\eta = b - z'\beta$ map to quantiles of the bid distribution.

The following assumptions underlie the homogenizing procedure.

AS7 Single index assumption: $F_1(\cdot|z) = F_1(\cdot|z'\beta)$, $F_2(\cdot|\cdot, z) = F_2(\cdot|\cdot, z'\beta)$

Define the quantiles $\alpha_1 \equiv F_1(v_1|z'\beta)$, $\alpha_2 \equiv F_2(v_2|z'\beta)$, and $\alpha_s \equiv F_2(s(v_1, v_2)|z'\beta)$. Also, using the distribution of second-auction prices $J(\cdot)$, define $\tilde{\alpha}_2 \equiv J(v_2|z'\beta)$ and $\tilde{\alpha}_s \equiv J(s(v_1, v_2)|z'\beta)$. By definition, these α's are in [0,1]. According to Sklar's Theorem, there exists a bivariate copula $C(\cdot, \cdot)$ for any bivariate joint distribution $F(\cdot, \cdot)$ such that $F(v_1, v_2) = C(\alpha_1, \alpha_2)$.[21] Transforming bid data to quantiles and "pooling" them across different z assumes this copula is invariant to z:

AS8 Copula restriction

1. $C(\alpha_1, \alpha_2|z) = C(\alpha_1, \alpha_2)$
2. $C(\alpha_1, \alpha_s|z) = C(\alpha_1, \alpha_s)$

AS8.2 implicitly restricts the synergy function $s(\cdot, \cdot)$ to preserve this quantile relationship across different $z'\beta$.

Remark 1. Under assumptions AS7 and AS8, $C(\alpha_1, \tilde{\alpha}_2)$ and $C(\alpha_1, \tilde{\alpha}_s)$ are also invariant to z.

It directly follows that the objects to be estimated - $\check{F}_2(\tilde{\alpha}_2|\alpha_1)$ and $\check{D}(\tilde{\alpha}_s|\alpha_1)$ - are invariant to $z'\beta$, since $\check{F}_2(\tilde{\alpha}_2|\alpha_1)$ is just a conditional of $C(\alpha_1, \tilde{\alpha}_2)$, for instance. Therefore, observations with different $z'\beta$ can be pooled in the first step of estimation once the bids and prices have been transformed to quantiles.

[20]Specifically, I estimate $\tilde{b} = \hat{G}(b|z'\beta)$ and $\tilde{p} = \hat{J}(p|z'\beta)$ using the empirical cdf of b and p, respectively, conditioning on $z'\beta$ via Epanechnikov kernels.

[21]I do not use a specific parametric form for the copula; my copula is nonparametric.

If I were to take the demeaning approach, I would need all of the assumptions made here and two more: that v_1, v_2, and $s(v_1, v_2)$ are additively separable in $z'\beta$ and a residual ϵ, and that these residuals have the same distribution regardless of $z'\beta$.[22] The quantile approach, on the other hand, does not assume additive separability and allows marginal distributions to vary with $z'\beta$. Besides avoiding strong assumptions, allowing flexibility in how $z'\beta$ affects values also allows covariates to absorb as much of the correlation between values as possible, making the efficiency tradeoff relative to demeaning worthwhile given the paper's goal of disentangling affiliation.

6 Estimation using the paired leases in New Mexico

6.1 Sample and model used

Since both winners and losers are needed in order to identify synergy, the number of bidders N must be at least 2. Also, since bid functions in A1 depend on N, a number of steps in the estimation procedure are conducted separately for each value of N. Therefore, there must be a sufficient number of auctions observed in the sample for each value of N considered in the estimation. Looking at Table 1, the sample size is largest at $N = 2$ and $N = 3$, and the number of observations becomes rather small for auctions with $N \geq 4$. Therefore, I use $N = 2$ and $N = 3$ in my estimation, which gives a sample of roughly 400 pairs. As the model assumes that the set of bidders participating in the first and second auction are the same, observations in which the second-auction winner did not bid in the first auction are dropped from the sieve maximum likelihood estimation.

The SLO employs a very longstanding, publicly known reserve price of roughly \$15.625 per acre. I assume this is non-binding; the agency considers the reserve price a "starting point" for serious bidders and tries not to offer tracts for which it might be binding. Meanwhile, as Kong (2020) found risk aversion to be important in the New Mexico oil and gas lease auctions, I allow bidders to be risk-averse. Thus the primitives of the model are the v_1-distribution $F_1(\cdot)$, the conditional v_2-distribution $F_2(\cdot|v_1)$, the synergy function $s(\cdot, \cdot)$, and the utility function $U(\cdot)$. I choose the

[22] I show in Online Appendix A.1 that Haile et al. (2003)'s method can be used for bid homogenization under those assumptions.

constant relative risk aversion (CRRA) specification for utility, so $U(\cdot)$ is represented by a risk aversion parameter ρ.

6.2 Covariates z

As discussed in section 5.2, the vector of lease characteristics z will be used to form a single index $z'\beta$ that represents the heterogeneity across leases. This section lists the z's used to form $z'\beta$. It should be emphasized that the main purpose here is not to study the β coefficient on each variable z. Rather, it is to form an index $z'\beta$ that absorbs as much of the heterogeneity between auction items as possible, such that conditional on the index, remaining variation in bidders' values is due to bidder-level idiosyncracies. The z's are chosen with this purpose in mind.

Observable characteristics of auctioned leases fall into three categories: lease terms, time of auction (industry, economic, local conditions of that time), and location of the tract (encompassing geological features). The royalty rate is indicated by the lease prefix, VA (subregular), V0 (regular), or VB (premium) in this sample; better tracts are assigned higher rates. As the VA prefix was discontinued in 2005, prefixes pre-2005 will be distinguished from prefixes post-2005. The contracted duration of a lease absent production does not vary in this sample, at 5 years.

To represent the effects of time, I include year fixed effects as well as month fixed effects to reflect any seasonality. These are supplemented by oil prices (West Texas Intermediate) and gas prices (natural gas 1 month futures). In addition, average price per acre in the previous month's auctions and average price per acre in the federal Bureau of Land Management's[23] lease sales in the same quarter are included to reflect local and industry conditions around the time.

The location of the tract implies geological information. As a first-level control, the volume of oil produced on the tract between 1970 and the auction date and the volume of oil produced after the auction date through 2014 are included as indicators of a tract's potential for production. I also construct and include as a covariate a smooth, location-based value index as follows. I take deflated sealed bid data from the SLO auctions and fit a smooth surface of these bids on geographic (north-south and east-west) coordinates using local quadratic regression. This procedure is performed once for each auction, excluding own-auction bids from the smoothing procedure,

[23] The BLM is a bureau that manages federal public lands, and is distinct from the State Land Office that manages state trust lands. Their auctions are quarterly.

and the index for each tract is the value predicted by the fitted surface excluding own-auction bids. This "heatmap" index serves as a further control for location-determined heterogeneity.

To form $z'\beta$, I regress log submitted sealed bids on these covariates. The regression coefficients are displayed in Table 10 of Online Appendix A.5. The heatmap index in particular has good explanatory power; a 1% increase in the index is associated with a 0.7% increase in bids. The coefficients on the premium lease prefix dummy, natural gas prices, and prices in recent lease sales are also positive and statistically significant.

6.3 Empirical results

In this section I discuss the empirical results from applying the estimation procedure to the data, starting with the estimated joint distribution of (v_1, v_2). I find that bidders with a high v_1 are more likely to have a high v_2; fitting a Kendall's tau to values simulated from $\hat{F}(v_1, v_2)$ yields 0.37. To place this value in context, simulations show that this amount of affiliation alone will cause the same bidder to win both objects 69% of the time in a two-bidder auction.

Figure 3 plots the estimated synergy function as a function of v_2 at different values of v_1. When reading the plot, it helps to know that the 90th percentile of v_2 is roughly \$85,000; i.e. data is concentrated in the left side of the plot. A dotted 45° line, representing zero synergy, is included for reference. The estimated synergy function lies above the 45° line, meaning $s(v_1, v_2) > v_2$ and synergy is positive. The lower bound of the 95% bootstrap percentile interval is also above the 45° line where the data is concentrated, indicating statistically significant synergy. For higher values of v_2, the added benefit of synergy, $s(v_1, v_2) - v_2$, appears fairly constant. This suggests that most of the synergy comes from fixed cost savings that "max out" at some value. For the median value of v_1 at median $z'\beta$ (about \$35000), $E_{v_2}[s(v_1, v_2) - v_2]$ is estimated to be on the order of \$13,000.

The SLO does allow leases to be transferred between firms, so the valuations that form v_1 and v_2 may already account for the possibility of buying (selling) synergistic leases from (to) other firms. Positive estimated synergy indicates and measures the fact that, even if v_2 already has this possibility built into its value, winning the first auction still increases bidders' values for the second lease. An important reason for this is that lease transfers must be negotiated between parties, so such a transaction is not frictionless.

Finally, the CRRA parameter ρ is estimated using the minimum-distance estimator defined by (9). The estimator's minimand is uniquely minimized by $\hat{\rho} = 0.47$,[24] indicating risk-averse bidders. A parameter value of $\rho = 0$ indicates risk-neutrality and $\rho = 1$ indicates log-utility. As a comparison, Holt and Laury (2002) measure CRRA parameters centered around the 0.3-0.5 range in laboratory experiments, and Lu and Perrigne (2008) measure roughly 0.59 for the USFS timber auctions.

I estimate a number of alternative specifications in Online Appendix A.6. First, I restrict the estimation sample to auctions with two bidders only. This is a subsample for which the evidence in Table 2 and A2 prices above the minimum acceptable bid provide sufficient confidence that A1 and A2 typically share the same bidders.[25] Second, I condition $F_2(\cdot|\cdot)$ and $D(\cdot|\cdot)$ on the number of auctions a bidder wins between A1 and A2, w; that is, I estimate $F_2(\cdot|v_1, w)$ and $D(\cdot|v_1, w)$ and the resulting synergy function. Third, rather than excluding observations in which the A2 winner did not bid in A1, I include these observations and estimate a model in which there is always one additional bidder in A2 than there is in A1. Synergy estimates in these alternative specifications are very close to the one shown in Figure 3 for values of v_2 below the 90th percentile, where data is most dense and bootstrap intervals are most narrow.

7 Counterfactuals

7.1 Simulated model

Using the structural estimates obtained from the estimation procedure, I first perform counterfactual simulations to understand the driving forces behind what I observe in the data. Table 5 displays the results.

The "observed" row shows what is observed in the data for pairs with $N = 2$ at median $z'\beta$.[26] Under the "simulated" heading, row (1) displays the expected revenue simulated using the full model; it is the simulated analog of the "observed" row. Subsequent rows show what revenue would be if selected elements of the full model

[24]See Online Appendix A.5 Figure 10.

[25]The $N = 3$ sample does not share this property because there is no way to disprove attrition, i.e. from three bidders in A1 to two bidders in A2.

[26]Revenue "at" median $z'\beta$ is computed via kernel regression of revenue on $z'\beta$.

were shut down.[27] Row (4), which simulates a sequence of two English auctions, will be useful later because it allows a comparison of sequential and bundled auctions holding the auction format fixed at a single format.

Comparing the "observed" row to simulated row (1) gives a sense of model fit. In addition, Figure 4 compares the empirical cdf of A1 bids and A2 prices in the actual data to the empirical cdf of those bids and prices simulated from the estimated model. The model does a good job of fitting the probability that the same bidder wins both tracts, expected revenue, and the distribution of bids and prices.

Section 2 documented that the A1-winner is more likely than other bidders to win A2, but until now I was unable to assess how much of this was due to synergy versus affiliation. Comparing rows (1) and (3) reveals that if synergy were eliminated, the proportion of cases in which the same bidder wins both tracts would drop from 75% to 69%. On the other hand, row (2) shows that if v_1 and v_2 were not affiliated, that percentage would drop sharply to 55%. I conclude that both synergy and affiliation are responsible for the same-winner phenomenon, but affiliation is the primary explanation. This highlights the importance of allowing for and distinguishing affiliation from synergy.

7.2 Bundled auctions

A policy alternative in the presence of synergy would be to auction the pair as a bundle, as this guarantees the winning bidder will realize synergy. A downside of bundling is that it forces a single bidder to take both tracts, even when the highest-value bidder for each tract is different. A general theoretical comparison of sequential versus bundled auctions that applies to this model does not exist. Having used Table 5 to understand the forces at work, I focus on the question of whether to bundle in Table 6. In this table, the auction format is held fixed at the English auction format to assess the effect of bundling without confounding factors. Column (a) simulates a sequence of two English auctions; i.e. Table 6's (1)(a) equals Table 5's (4)(c). Columns (b) and (c) simulate bundled auctions under two different informational assumptions: one in which the bidder bids based on bundle value $v_1 + s(v_1, v_2)$ and

[27] In row (2), which simulates the counterfactual scenario of no affiliation, pairs of (v_1, v_2) emerge in simulation that are rarely observed in the data, e.g. very low v_1 and very high v_2. For these extreme draws, $s(v_1, v_2)$ is not well identified. Since $s(v_1, v_2)$ does not change dramatically with v_1 according to Figure 3, I do the following for row (2) simulations: compute the synergy-inclusive value of the second lease as a function of v_2 only, i.e. $s(v_1, v_2) = \check{s}(v_2) \equiv D^{-1}(F_2(v_2))$.

the other in which the bidder is uncertain about v_2 and bids based on an expected value for the bundle, $v_1 + \mathbb{E}[s(v_1, v_2)|v_1]$.

In both the two-bidder and three-bidder case (rows (1) and (3), respectively), revenues in columns (b) and (c) are higher than in (a); the effect of bundling on auction revenue is positive. The increase amount depends on the level of bidders' certainty regarding v_2, but it is at least 7%. As shown in Table 1, a large majority of these auctions receive three bids or less. The finding that bundling would be better for revenue is consistent with the computations of Subramaniam and Venkatesh (2009), which suggest that the smaller the number of bidders, the more likely are bundled auctions to dominate sequential auctions in terms of revenue. This can be reversed for larger N, where it may be optimal to exploit competition twice by selling each tract separately. The result is also generally consistent with papers that study bundling in contexts without synergy, such as Palfrey (1983) and Chakraborty (1999). Both of these papers find that the smaller the number of bidders, the more likely is bundling to increase revenue in Vickrey auctions.

Meanwhile, since these auctions are run by a public institution, revenue considerations must be balanced against allocative efficiency, or the desire to award tracts to the firms that value them most. Rows (2) and (4) address allocative efficiency by computing the total value derived from a pair of tracts by the winner(s). If a single bidder wins both - which is always the case for bundled auctions - this total value is inclusive of synergy. The table shows that bundling leads to a loss in this total value of 2-3% in the informational case of column (b) and more in the case of column (c). While bundling guarantees that synergy will be realized, it gives up the flexibility of allocating the two leases to different bidders. On net the negative effect of the latter on efficiency outweighs the positive effect of the former in these auctions.

One caveat in interpreting these results is that this comparison of sequential versus bundled auctions holds the number of bidders constant across the two policies. Leases of the bundled size are too rare in the data to deduce whether and how the act of bundling would change the number of bidders. Here I provide computations for the baseline case of no change.

7.3 Generalized bundling

As a primary policy question, I considered bundling two leases, or doubling the area covered by a lease. Tracts cannot get smaller than their current size because, by state rules, wells producing gas must be allotted at least 320 acres. Nonetheless, I secondarily consider the general question of how auctioning a single large tract compares to splitting it into T pieces and auctioning them sequentially. There are some caveats to this exercise. First, the structure of synergy estimated between two 320-acre tracts need not extrapolate to other sizes. Second, derivation of a full equilibrium for $T > 2$ is not trivial, and I leave this for future research. Here, I look for qualitative answers by simulating longer auction sequences under the following assumptions: (1) synergy exists only between adjacent tracts in the sequence; (2) when bidding in auction t, bidders account for the direct effect of winning auction t on auction $t+1$ but ignore ripple effects on auctions $t+2$, ..., T ; (3) when bidding in auction t, bidders do not use their memory of past auctions $t-1$, $t-2$, etc. to predict future competition in auction $t+1$; (4) $F_{t+1}(v_{t+1}|v_t) = F_2(v_{t+1}|v_t)$ and $s_t(v_t, v_{t+1}) = s(v_t, v_{t+1})$.

Then at every auction t other than the last one, bidders bid according to

$$b_t(v_t, v_t') = v_t' + \underbrace{\int_{v_{t+1}=\underline{v}}^{\bar{v}} \left\{ \int_{u=\underline{v}}^{s(v_t,v_{t+1})} \tilde{H}_1(u|v_t)du - \int_{u=\underline{v}}^{v_{t+1}} \tilde{H}_2(u|v_t)du \right\} dF_2(v_{t+1}|v_t)}_{\text{expected benefit in auction } t+1 \text{ of winning auction } t}.$$

where $v_t' = \begin{cases} v_t & \text{if } t = 1 \text{ or lost auction } t-1 \\ s(v_{t-1}, v_t) & \text{if won auction } t-1 \end{cases}$. In auction T, bidders bid v_T'.

Table 7 presents simulation outcomes for a 3-auction and 4-auction sequence. Comparing these sequential auctions to a single bundle of all tracts gives us a sense of how revenue and efficiency respond to breaking a given tract into more and more pieces. A bundle of 3 leases revenue-dominates sequential auctions by 16%, and this increases to 23% for 4 leases. On the other hand, bundling causes a 5% and 7% decrease in the measure of allocative efficiency relative to 3-auction and 4-auction sequences, respectively. The pattern emerging from this table, combined with Table

6, is that revenue here declines as a function of the number of pieces a given piece of land is split into, but allocative efficiency increases.

8 Conclusion

This paper performs a structural analysis of auctions that take place sequentially, are linked by synergy, and in which each bidder's values can be affiliated across auctions. It explains that ignoring affiliation can lead to exaggerated estimates of synergy and distinguishes synergy from affiliation in identifying and estimating the auction model. The model uses general functional forms for synergy and the joint distribution of values while maintaining tractability in equilibrium analysis. The paper establishes nonparametric identification of the model and develops a multi-step estimation procedure that recovers all model primitives. Applying the estimation method to New Mexico oil and gas lease data, I find both synergy and affiliation between adjacent tracts. Affiliation is more important than synergy when it comes to explaining why the same bidder often wins both tracts. Counterfactual simulations predict that bundled auctions would yield higher revenue than sequential auctions given the estimated combination of synergy, affiliation, and the typically low number of bidders.

The paper opens the door to distinguishing causal effects from persistent heterogeneity in other types of sequential auctions or sequential events that generate bids in the first stage. Its main insight for doing so is adaptable to other sequences as long as first-auction bids are monotonic in values and observed. Another very interesting possibility is that of extending the method to a longer sequence of affiliated items. As alluded to in section 7.3, affiliation of a bidder's values across a longer sequence creates additional challenges for equilibrium derivation, but the structure of this model offers hope for analysis with the help of some well-placed assumptions. Finally, the paper's insights may be useful for studying collusion in the context of sequential auctions, which, relative to bundled auctions, alter the incentives for and sustainability of collusive agreements. These questions remain open for future research.

References

Agarwal, S., J. Li, E. Teo, and A. Cheong (2017): "Strategic Sequential Bidding for Government Land Auction Sales – Evidence from Singapore," *Journal of Real Estate Finance and Economics*, 1–31.

Ashenfelter, O. (1989): "How Auctions Work for Wine and Art," *Journal of Economic Perspectives*, 3, 23–36.

Athey, S. and P. A. Haile (2002): "Identification of Standard Auction Models," *Econometrica*, 70, 2107–2140.

Ausubel, L. M., P. Cramton, R. P. McAfee, and J. McMillan (1997): "Synergies in Wireless Telephony: Evidence from the Broadband PCS Auctions," *Journal of Economics and Management Strategy*, 6, 497–527.

Balat, J. (2016): "Identification and Estimation of Affiliated Private Values Auctions with Unobserved Heterogeneity," https://sites.google.com/site/jbalat/research.

——— (2017): "Highway Procurement and the Stimulus Package: Identification and Estimation of Dynamic Auctions with Unobserved Heterogeneity," https://sites.google.com/site/jbalat/research.

Benoit, J.-P. and V. Krishna (2001): "Multiple-Object Auctions with Budget Constrained Bidders," *Review of Economic Studies*, 68, 155–179.

Bierens, H. J. and H. Song (2012): "Semi-Nonparametric Estimation of Independently and Identically Repeated First-Price Auctions via an Integrated Simulated Moments Method," *Journal of Econometrics*, 168, 108–119.

Brendstrup, B. and H. J. Paarsch (2006): "Identification and Estimation in Sequential, Asymmetric, English Auctions," *Journal of Econometrics*, 134, 69–94.

Caillaud, B. and C. Mezzetti (2004): "Equilibrium Reserve Prices in Sequential Ascending Auctions," *Journal of Economic Theory*, 117, 78–95.

Calonico, S., M. D. Cattaneo, M. H. Farrell, and R. Titiunik (2018): "Regression Discontinuity Designs Using Covariates," https://sites.google.com/site/rdpackages/rdrobust.

Calonico, S., M. D. Cattaneo, and R. Titiunik (2014a): "Robust Data-Driven Inference in the Regression-Discontinuity Design," *Stata Journal*, 14, 909–946.

——— (2014b): "Robust Nonparametric Confidence Intervals for Regression-Discontinuity Designs," *Econometrica*, 82, 2295–2326.

Cantillon, E. and M. Pesendorfer (2006): "Combination Bidding in Multi-Unit Auctions," Available at SSRN 958730.

Chakraborty, I. (1999): "Bundling Decisions for Selling Multiple Objects," *Economic Theory*, 13, 723–733.

Chen, X. (2007): "Large Sample Sieve Estimation of Semi-Nonparametric Models," in *Handbook of Econometrics*, ed. by J. J. Heckman and E. E. Leamer, Elsevier, vol. 6B, 5549–5632.

Colwell, P. F. and A. Yavaş (1994): "The Demand for Agricultural Land and Strategic Bidding in Auctions," *Journal of Real Estate Finance and Economics*, 8, 137–149.

De Silva, D. G., T. D. Jeitschko, and G. Kosmopoulou (2005): "Stochastic Synergies in Sequential Auctions," *International Journal of Industrial Organization*, 23, 183–201.

Donald, S. G., H. J. Paarsch, and J. Robert (2006): "An Empirical Model of the Multi-Unit, Sequential, Clock Auction," *Journal of Applied Econometrics*, 21, 1221–1247.

Donna, J. D. and J.-A. Espín-Sánchez (2018): "Complements and Substitutes in Sequential Auctions: The Case of Water Auctions," *RAND Journal of Economics*, 49, 87–127.

Fox, J. T. and P. Bajari (2013): "Measuring the Efficiency of an FCC Spectrum Auction," *American Economic Journal: Microeconomics*, 5, 100–146.

Gandal, N. (1997): "Sequential Auctions of Interdependent Objects: Israeli Cable TV Licenses," *Journal of Industrial Economics*, 45, 227–244.

GENTRY, M. L., T. KOMAROVA, AND P. SCHIRALDI (2016): "Preferences and Performance in Simultaneous First-Price Auctions: A Structural Analysis," Available at SSRN 2514995.

GROEGER, J. R. (2014): "A Study of Participation in Dynamic Auctions," *International Economic Review*, 55, 1129–1154.

GUERRE, E., I. PERRIGNE, AND Q. VUONG (2000): "Optimal Nonparametric Estimation of First-Price Auctions," *Econometrica*, 68, 525–574.

HAHN, J., P. TODD, AND W. VAN DER KLAAUW (2001): "Identification and Estimation of Treatment Effects with a Regression-Discontinuity Design," *Econometrica*, 69, 201–209.

HAILE, P. A., H. HONG, AND M. SHUM (2003): "Nonparametric Tests for Common Values at First-Price Sealed-Bid Auctions," NBER Working Paper No. 10105.

HAUSCH, D. B. (1986): "Multi-Object Auctions: Sequential vs. Simultaneous Sales," *Management Science*, 32, 1599–1610.

HECKMAN, J. J. (1981): "Heterogeneity and State Dependence," in *Studies in labor markets*, ed. by S. Rosen, University of Chicago Press, 91–140.

HOLT, C. A. AND S. K. LAURY (2002): "Risk Aversion and Incentive Effects," *American Economic Review*, 92, 1644–1655.

HUBBARD, T. P., T. LI, AND H. J. PAARSCH (2012): "Semiparametric Estimation in Models of First-Price, Sealed-Bid Auctions with Affiliation," *Journal of Econometrics*, 168, 4–16.

JOFRE-BONET, M. AND M. PESENDORFER (2003): "Estimation of a Dynamic Auction Game," *Econometrica*, 71, 1443–1489.

KATZMAN, B. (1999): "A Two Stage Sequential Auction with Multi-Unit Demands," *Journal of Economic Theory*, 86, 77–99.

KAWAI, K. AND J. NAKABAYASHI (2014): "Detecting Large-Scale Collusion in Procurement Auctions," Available at SSRN 2467175.

Komarova, T. (2017): "Extremum Sieve Estimation in k-out-of-n Systems," *Communications in Statistics - Theory and Methods*, 46, 4915–4931.

Kong, Y. (2020): "Not Knowing the Competition: Evidence and Implications for Auction Design," *RAND Journal of Economics*, forthcoming.

Laffont, J.-J. and Q. Vuong (1996): "Structural Analysis of Auction Data," *American Economic Review, Papers and Proceedings*, 86, 414–420.

Lamy, L. (2010): "Identification and Estimation of Sequential English Auctions," PSE Working Papers n2010-16, halshs-00564887.

——— (2012): "Equilibria in Two-Stage Sequential Second-Price Auctions with Multi-Unit Demands," https://sites.google.com/site/laurentlamy78/articles.

Li, T., I. Perrigne, and Q. Vuong (2002): "Structural Estimation of the Affiliated Private Value Auction Model," *RAND Journal of Economics*, 171–193.

Li, T. and B. Zhang (2015): "Affiliation and Entry in First-Price Auctions with Heterogeneous Bidders: An Analysis of Merger Effects," *American Economic Journal: Microeconomics*, 7, 188–214.

Lu, J. and I. Perrigne (2008): "Estimating Risk Aversion from Ascending and Sealed-Bid Auctions: The Case of Timber Auction Data," *Journal of Applied Econometrics*, 23, 871–896.

Marshall, R. C., M. E. Raiff, J.-F. Richard, and S. P. Schulenberg (2006): "The Impact of Delivery Synergies on Bidding in the Georgia School Milk Market," *Topics in Economic Analysis & Policy*, 6, Article 5.

Mes, M. (2008): "Sequential Auctions for Full Truckload Allocation," Ph.D. thesis, University of Twente, Enschede, the Netherlands.

Milgrom, P. R. and R. J. Weber (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.

Moreton, P. S. and P. T. Spiller (1998): "What's in the Air: Interlicense Synergies in the Federal Communications Commission's Broadband Personal Communication Service Spectrum Auctions," *Journal of Law & Economics*, 41, 677–716.

Ortega-Reichert, A. (1968): "Models for Competitive Bidding Under Uncertainty," Department of Operations Research Technical Report 8, Stanford University, Stanford, CA.

Palfrey, T. R. (1983): "Bundling Decisions by a Multiproduct Monopolist with Incomplete Information," *Econometrica*, 51, 463–483.

Pinkse, J. and G. Tan (2005): "The Affiliation Effect in First-Price Auctions," *Econometrica*, 73, 263–277.

Reny, P. J. and S. Zamir (2004): "On the Existence of Pure Strategy Monotone Equilibria in Asymmetric First-Price Auctions," *Econometrica*, 72, 1105–1125.

Riley, J. G. and W. F. Samuelson (1981): "Optimal Auctions," *American Economic Review*, 71, 381–392.

Schmalensee, R. (1984): "Gaussian Demand and Commodity Bundling," *Journal of Business*, 57, S211–S230.

Somaini, P. (2018): "Identification in Auction Models with Interdependent Costs," Research Papers 3702, Stanford University, Graduate School of Business.

Subramaniam, R. and R. Venkatesh (2009): "Optimal Bundling Strategies in Multiobject Auctions of Complements or Substitutes," *Marketing Science*, 28, 264–273.

Sunnevåg, K. J. (2000): "Designing Auctions for Offshore Petroleum Lease Allocation," *Resources Policy*, 26, 3–16.

Table 1: Number of pairs, by number of bidders N in the first auction

N	0	1	2	3	4	5	6	7	8
pairs	14	267	247	165	98	50	21	9	1

Table 2: Statistics for paired leases

For $N \geq 2$:		
Correlation of final price in 1st and 2nd auction	0.91	
Probability that 2nd-auction winner also bid on 1st auction	93%	
Probability that pair is won by same bidder:	observed	even odds
$N = 2$	74%	50%
$N = 3$	62%	33%
$N = 4$	67%	25%

Table 3: Sharp RD estimates using local linear regression

	(1)	(2)	(3)
Conventional	0.208	0.226	0.221
	(0.101)	(0.102)	(0.105)
Bias-corrected	0.197	0.209	0.204
	(0.101)	(0.102)	(0.105)
Robust	0.197	0.209	0.204
	(0.124)	(0.124)	(0.126)
Fixed effects for $N = 2$ and $N = 3$	N	Y	Y
Interactions $z \times (N = 2)$ and $z \times (N = 3)$	N	N	Y
Observations	543	543	543

Epanechnikov kernel. Bandwidth selection according to Calonico et al. (2014b) and Calonico et al. (2018) using `rdrobust` package. Standard errors in parentheses.

Table 4: Probit regression

	Win 2nd auction
Won 1st auction	0.696
	(0.125)
z	0.864
	(0.134)
z*(number of auctions between A1 and A2)	-0.009
	(0.003)
Number of bidders fixed effects	Y
Bidder fixed effects	Y
Observations	1557

Standard errors in parentheses

Table 5: Counterfactual revenue for pair

		% same winner	(a) A1	(b) A2	(c) Total
Observed		78%*	52,682	40,155	92,837
Simulated					
(1)	S + RA + A	75%	53,669	39,082	92,751
(2)	S + RA	55%	50,865	39,645	90,510
(3)	RA + A	69%	45,469	36,507	81,976
(4)	EE: S + A	75%	44,999	39,082	84,081

*Excluding cases where A2-winner did not bid in A1
"S" = synergy, "A" = affiliation, "RA" = risk aversion,
"EE" = sequence of two English auctions, risk neutral

Table 6: Sequential versus bundled auctions

(English auctions, risk neutral)		Sequential	Bundled auctions		
		(a)	(b)	(c)	
			$v_1 + s(v_1, v_2)$	$v_1 + \mathbb{E}[s(v_1, v_2)\|v_1]$	$\frac{\text{(b)-(a)}}{\text{(a)}}$
$N = 2$					
(1)	Revenue per pair	84,081	89,571	96,714	7%
(2)	Value of tracts to winner(s)	349,070	343,609	336,478	-2%
$N = 3$					
(3)	Revenue per pair	136,621	146,417	146,896	7%
(4)	Value of tracts to winner(s)	423,869	410,250	398,303	-3%

At median $z'\beta$

Table 7: Longer sequences versus bundled auctions

(English auctions, risk neutral)	Sequential auctions					Bundled	
$N = 2$	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	A1	A2	A3	A4	Total		$\frac{\text{(f)-(e)}}{\text{(e)}}$
3 leases							
(1) Revenue	44,999	49,299	39,903	-	134,201	156,277	16%
(2) Value of tracts to winner(s)			466,649			444,680	-5%
4 leases							
(3) Revenue	44,999	49,299	50,118	39,267	183,682	226,106	23%
(4) Value of tracts to winner(s)			586,018			543,693	-7%

At median $z'\beta$. The value of the bundle is $v_1 + s(v_1, v_2) + s(v_2, v_3)$ for 3 leases and $v_1 + s(v_1, v_2) + s(v_2, v_3) + s(v_3, v_4)$ for 4 leases.

Figure 1: Regression discontinuity plot

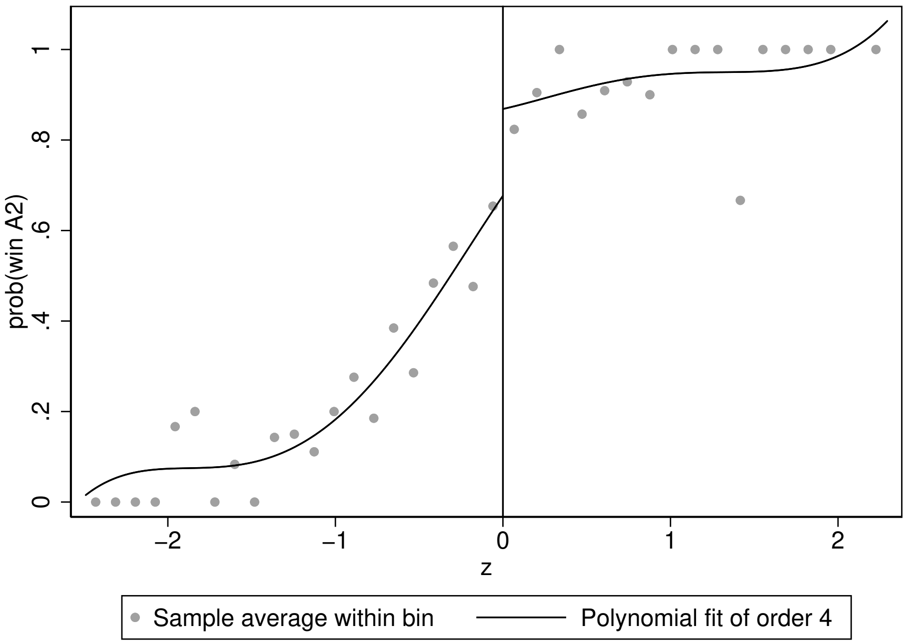


Figure 2: Nonparametric identification of synergy function

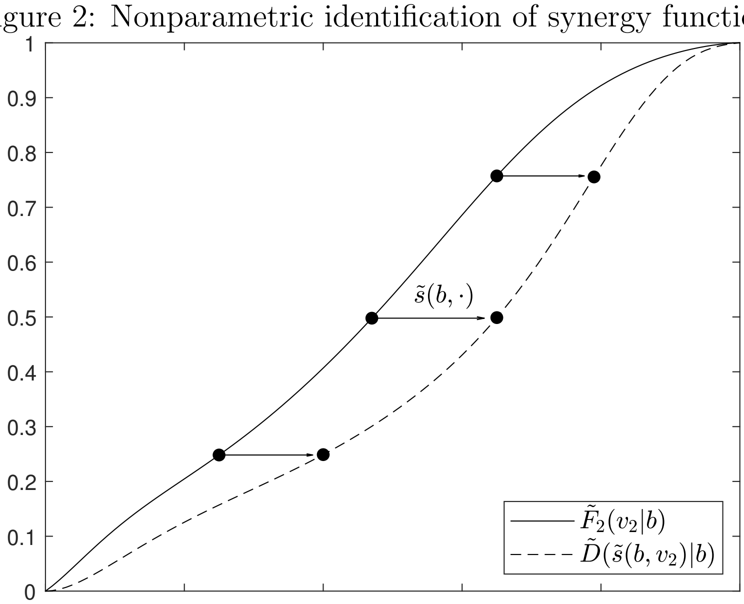
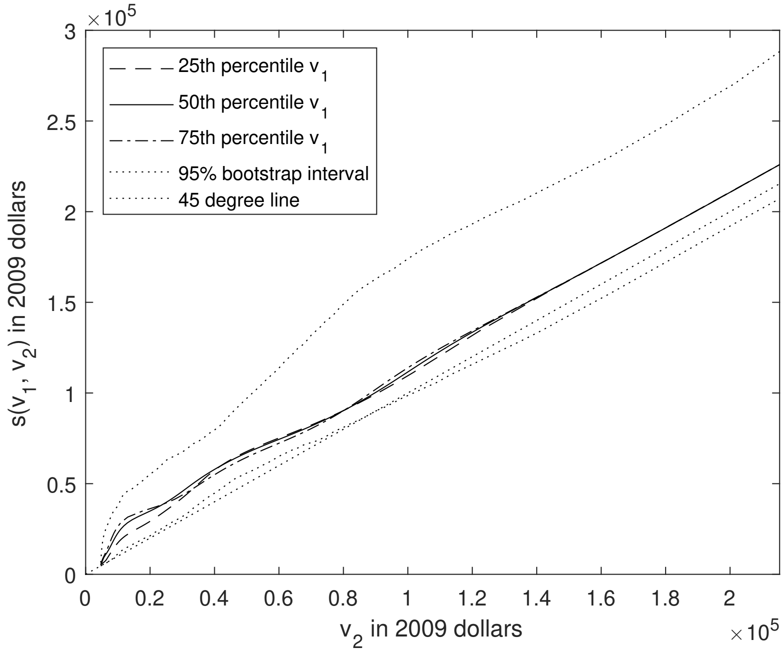
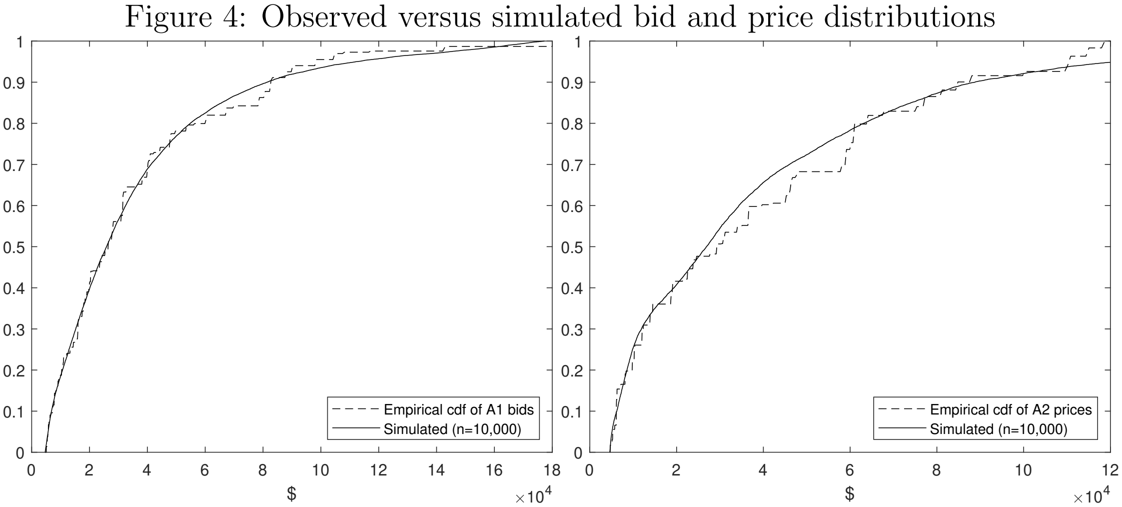


Figure 3: Estimated synergy function



The black dotted lines show the 95% bootstrap percentile interval for the black solid line, which is $\hat{s}(v_1, v_2)$ evaluated at the 50th percentile of v_1.

Figure 4: Observed versus simulated bid and price distributions



$N = 2$, median $z'\beta$